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DETERMINATION OF THE CHARGE
OF RELATIVISTIC HEAVY NUCLEI
FROM EMULSION TRACKS

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION



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ABSTRACT

The number of δ rays with energies between 50 and 150 keV that are produced by heavy nuclei in emulsions is calculated. The Z^2 dependence predicted by the first Born approximation is corrected by a direct calculation of the Mott exact phase-shift scattering cross section. Comparisons are made with corrections predicted by the second Born approximation. When the phase-shift results are applied to the problem of charge identification, corrections of up to 4 units of charge for 1.457-GeV/nucleon nuclei with charge $Z = 75$ are found.

INTRODUCTION

Fowler et al. have recently published results on the detection of heavy nuclei in cosmic rays with charge $Z > 64$ with an estimated accuracy of 2 per-cent at the geomagnetic cut-off energy of the experiment [1]. The analysis of the delta rays produced in the emulsion was based upon the first Born approximation. Corrections to the Z^2 dependence predicted by the first Born approximation were made on the basis of published results [2] of the ratio of the Mott-to-Rutherford scattering cross section at the minimum observed δ -ray energy.

Since the first Born approximation condition $Z/137\beta \ll 1$ is not fulfilled for large values of Z , Semikoz suggested that a reduction in the reported charge estimate by Fowler was necessary [3]. He based his results on a calculation utilizing the second Born approximation.

The purpose of this paper is to present an investigation of the determination of the charge of heavy nuclei using the Mott phase-shift calculation for the scattering cross section. After this cross section is calculated, a numerical integration is performed over the delta-ray energy range to obtain the number of delta rays produced per unit path length of the heavy nucleus. These results are then compared to those obtained using the first and second Born approximation.

THEORY

Because only those delta rays with energies much greater than the binding energies of the emulsion atomic electrons are used for charge identification, one may assume that the atomic electrons are initially free and at rest. Thus, by a transformation to the frame in which the electron is initially at rest, the cross section for delta ray production $d\sigma/dT$ may be obtained from the Mott cross section $d\sigma/d\Omega$ normally used for the scattering of an electron from a point nucleus. One then finds the cross sections transform as

$$\frac{d\sigma}{dT} = \frac{4\pi}{T_{\max}} \frac{d\sigma}{d\Omega},$$

where T , the energy transfer to the electron, and T_{\max} , the maximum energy transferable to an electron, are related to the electron-scattering angle in the cross section differential with respect to solid angle by the relation $\sin^2 \theta/2 = T/T_{\max}$. If the nucleus is considered infinitely heavy in comparison with the electron, then $T_{\max} \approx 2 m_e c^2 \beta^2 (1 - \beta^2)^{-1}$.

In the first Born approximation, the cross section is

$$\left(\frac{d\sigma}{dT}\right)_{B1} = \frac{2\pi r_0^2 Z^2}{\beta^2 T^2} \left(1 - \beta^2 T/T_{\max}\right), \quad (1)$$

where r_0 is the classical electron radius and Ze is the charge of the incident nucleus.¹

In the second Born approximation, the result is

$$\left(\frac{d\sigma}{dT}\right)_{B2} = \frac{2\pi r_0^2 Z^2}{\beta^2 T^2} \left\{ 1 - \beta^2 T/T_{\max} + \pi \alpha Z \beta \left(\frac{T}{T_{\max}}\right)^{1/2} \left[1 - \left(\frac{T}{T_{\max}}\right)^{1/2}\right] \right\}, \quad (2)$$

where $\alpha = 1/137$ is the fine-structure constant.

Therefore, one has a Z^2 charge dependence in the first case with an additional term proportional to Z^3 in the latter case. However, since the expansion in (αZ) converges slowly, one should consider the higher-order terms.

An alternative approach is to use a phase-shift calculation. The cross section thus obtained for the case of a point coulomb potential is [4]:

$$\left(\frac{d\sigma}{dT}\right)_{PS} = \frac{4\pi r_0^2}{(\alpha\beta\gamma)^2} \left[|F'|^2 T^{-1} + |G|^2 (T_{\max} - T)^{-1} \right], \quad (3)$$

where

$$F' = iq F/\gamma, \quad F = F_0 + F_1, \quad G = G_0 + G_1$$

$$\gamma = (1 - \beta^2)^{-1/2}, \quad q = \alpha Z/\beta$$

$$F_0 = \frac{i}{2} \frac{\Gamma(1 - iq)}{\Gamma(1 + iq)} \exp \left[iq \ln (T/T_{\max}) \right]$$

1. Unless otherwise indicated, $m_e c^2$ units are used throughout this paper, where m_e is the rest mass of the electron and $\hbar = m_e c = 1$.

$$G_0 = -iq \left[(T_{\max} - T)/T \right] F_0$$

$$F_1 = \frac{i}{2} \sum_{\ell=0}^{\infty} A_{\ell} P_{\ell}(\mu)$$

$$G_1 = \frac{i}{2} \sum_{\ell=0}^{\infty} B_{\ell} P_{\ell}(\mu)$$

$$A_{\ell} = (-1)^{\ell} \left[\ell D_{\ell} + (\ell + 1) D_{\ell+1} \right]$$

$$B_{\ell} = (-1)^{\ell} \left[\ell^2 D_{\ell} - (\ell + 1)^2 D_{\ell+1} \right]$$

$$D_{\ell} = \frac{\exp(-i\pi\ell)}{(\ell + iq)} \frac{\Gamma(\ell - iq)}{\Gamma(\ell + iq)} - \frac{\exp(-i\pi\rho_{\ell})}{(\rho_{\ell} + iq)} \frac{\Gamma(\rho_{\ell} - iq)}{\Gamma(\rho_{\ell} + iq)}$$

$$\rho_{\ell} = \sqrt{\ell^2 - (\alpha Z)^2}$$

and

$$\mu = 1 - 2 T/T_{\max},$$

which is found by noting that $\sin^2 \theta/2 = T/T_{\max}$ implies that $\cos \theta = 1 - 2 T/T_{\max}$.

Two transformations were made on the F_1 and G_1 series to obtain more rapid convergence. First, to improve the convergence for small energy transfers, the reduced series employed by Sherman [4] with $m = 3$ was used. This transformation is

$$(1 - \mu)^m F_1 = \frac{i}{2} \sum_{\ell=0}^{\infty} A_{\ell}^{(m)} P_{\ell}(\mu),$$

where

$$A_{\ell}^{(m)} = A_{\ell}^{(m-1)} - \frac{\ell+1}{2\ell+3} A_{\ell+1}^{(m-1)} - \frac{\ell}{2\ell-1} A_{\ell-1}^{(m-1)},$$

and similarly for G_1 .

Next the Euler transformation, which in our case results in a more rapidly converging series, was made to improve the convergence for large energy transfers. This transformation is

$$\sum_{\ell=0}^{\infty} (-1)^{\ell} A_{\ell} = \sum_{k=0}^{\infty} \frac{1}{2^{k+1}} \sum_{n=0}^k (-1)^n \frac{k!}{(k-n)! n!} A_n ,$$

or, in our case,

$$F_1 = \frac{i}{2} \sum_{k=0}^{\infty} \frac{1}{2^{k+1}} \sum_{n=0}^k \frac{k!}{(k-n)! n!} \frac{A_n^{(m)} P_n(\mu)}{(1-\mu)^m}$$

$$G_1 = \frac{i}{2} \sum_{k=0}^{\infty} \frac{1}{2^{k+1}} \sum_{n=0}^k \frac{k!}{(k-n)! n!} \frac{B_n^{(m)} P_n(\mu)}{(1-\mu)^m} .$$

The number of delta rays produced with kinetic energies between T_1 and T_2 per centimeter of the cosmic-ray path is found by calculating

$$N = \eta \int_{T_1}^{T_2} \left(\frac{d\sigma}{dT} \right) dT , \quad (4)$$

where η is the number of atomic electrons per cubic centimeter of the emulsion. In the case of the first and second Born approximation, this calculation can be done analytically. However, the phase-shift result must be obtained numerically.

Following Semikoz, after the number of electrons produced is found from Equation (4), the comparison with the first Born approximation is made by computing [3]:

$$\Delta N = (N_i - N_{B1})/N_i , \quad (5)$$

where i implies either the second Born or phase-shift calculation and N_{B1} is the number of delta rays obtained by using the first Born approximation.

An estimate of the charge correction for results based upon the first Born approximation can then be found simply as

$$Z_R = Z (1 - \Delta N/2) \quad , \quad (6)$$

or $|\Delta Z| = |Z - Z_R|$, where Z is the charge found by using the first Born approximation.

CALCULATION PROCEDURE

To calculate the phase-shift formula, a computer code was developed. The approach that was used followed that of Sherman in his calculations of electron scattering from point nuclei and his results were duplicated [4]. In addition, the small-angle approximation of Bartlett and Watson was utilized when very small angles were required by the specific case being run [5].

Double precision arithmetic (a resolution to 18 significant digits for the UNIVAC 1108 system) was used throughout the calculation. Table 1 illustrates a sample test that was used to check convergence. A Newton-Coates technique was used to perform the numerical integration with a resulting error of less than 1 percent. The cross section values calculated are estimated to be accurate to 3 significant figures. Round-off error was estimated by comparing the numerically summed series on the left-hand side of equation (7) to the analytically equivalent one on the right-hand side [6],

$$\sum_{\ell=0}^{\infty} (2\ell+1) P_{\ell}(\mu) x^{\ell} = (1-x^2) (1-2\mu+x^2)^{-3/2} \quad , \quad (7)$$

with $x = e^{-0.01}$ and $\ell = 0$ to $\ell = 175$. Agreement was found to be within four significant figures.

Stieltjes' continued fraction was used to compute the gamma functions of complex argument [7]. The Legendre functions were computed by using a recursion formula with $P_0(\mu) = 1$ and $P_1(\mu) = \mu$.

Since a very large number of terms are required at small angles, Bartlett and Watson [5] derived an approximation to evaluate the Mott phase-shift formula as $\theta \rightarrow 0$. This expression transformed to the rest frame of the electron is given by:

$$\left(\frac{d\sigma}{dT}\right)_{SA} = \frac{2\pi r_0^2 Z^2}{\beta^2 T^2} \left[1 + \pi \alpha Z \left(\frac{T}{T_{max}}\right)^{1/2} \cos \gamma \right], \quad (8)$$

where

$$e^{i\gamma} = \frac{\Gamma(1/2 - i q)}{\Gamma(1/2 + i q)} \frac{\Gamma(1 + i q)}{\Gamma(1 - i q)}$$

or $\cos \gamma = \text{Re}(e^{i\gamma})$.

Whenever T/T_{max} corresponded to a small angle, the largest value of T for which

$$\left| \left[\left(\frac{d\sigma}{dT}\right)_{SA} - \left(\frac{d\sigma}{dT}\right)_{PS} \right] / \left(\frac{d\sigma}{dT}\right)_{PS} \right| \leq 0.01$$

was found. For all smaller values of T , the small-angle approximation was used, whereas for all larger values the regular phase-shift result was used. The two cross sections are shown in Figure 1 for the cases $Z = 26$ and $Z = 104$.

RESULTS

Figure 2 shows the ratios of the phase-shift cross section to the first and second Born approximations for $Z = 92$ with $\beta = 0.92$. It may be noted from the figure that the second Born is higher than the phase-shift result throughout this range of energy transfer. This is not the case for an iron nucleus, as shown in Figure 3.

Figure 4 is the plot of the ratio of the second Born to the Rutherford cross section and the phase-shift calculation to the Rutherford cross section as a function of Z for nuclei with a kinetic energy of 3.0 and 1.457 BeV/nucleon.² The energy transfer was assumed to be 50 keV. In the figure the ratio for the phase-shift calculation reaches a maximum at around $Z = 60$ for $\beta = 0.92$ and at $Z = 75$ for $\beta = 0.9712$, and then decreases at the higher Z nuclei. However, the ratio for the second Born approximation increases in proportion to Z as expected. Therefore, in the energy ranges that are considered here, the

2. To convert electron volts to SI units in joules, multiply by 1.60210×10^{-19} .

higher-order terms in (αZ) become important whenever (αZ) is greater than about $(50/137)$. The third Born approximation, i. e., retention of terms through $(\alpha Z)^4$, is not expected to improve the trend significantly, because the ratio $(d\sigma/d\Omega)_{\text{third Born}} / \sigma_{\text{Rutherford}}$ to the Rutherford cross section still continues to be higher than the exact phase-shift ratio to the Rutherford at small angles [8]. Figure 5 shows the results of the same calculations for an energy transfer of 150 keV.

Table 2 shows a comparison between the charge correction $|\Delta Z|$ to the first Born approximation obtained by using the second Born approximation and the phase-shift calculations in equations (4) through (6). The energy transfer range used in equation (4) was assumed to be 50 to 150 keV. One notes from the table that corrections predicted by using the phase-shift formula range from about 1 for iron to 4 units of charge for $Z = 75$ and an incident nucleus with a kinetic energy of 1.457 GeV/nucleon. The second Born approximation, although agreeing up to $Z \approx 52$, predicts a correction of 10 units of charge at $Z = 104$.

CONCLUSIONS

Because the series used to calculate the phase-shift formula is slowly and conditionally convergent, one possible source of error would be that convergence has not been achieved, especially for those energy transfers that correspond to very small angles. However, great care has been exercised in the calculations to obtain convergence, both with respect to including transformations to obtain a more rapidly converging series and by including a large number (100) of terms. In addition, convergence tests were made in which up to 250 terms were used.

One notes immediately from the preceding results that the use of the second Born approximation, in the energy transfer region under consideration, leads to a gross overestimation in the correction to the Z^2 dependence of very highly charged nuclei producing knock-on electrons.

One also finds the largest correction in charge determination due to the departure of the scattering cross section from a Z^2 dependence to be approximately 4 units of charge at a kinetic energy of 1.457 GeV/nucleon ($\beta = 0.92$). This is slightly higher than that quoted by Fowler [1] and much lower than the correction given by Semikoz [3].

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TABLE 1. CONVERGENCE TESTS FOR F_1 , G_1 , AND $d\sigma/dT$
 (T_0 is the incident nuclei kinetic energy in units of GeV/nucleon.
 $d\sigma/dT$ is in units of Barns/MeV .)

a. $Z = 92 - \beta = 0.90$ ($T_0 = 1.216$)						
No Terms	$T = 50$ keV			$T = 150$ keV		
	30	100	250	30	100	250
Re F_1	0.02434	0.02381	0.02381	0.15859	0.15509	0.15509
Im F_1	-0.16202	-0.05584	-0.05583	0.02239	0.01397	0.01397
Re G_1	-5.40927	-6.98770	-6.98808	-2.53794	-2.53166	-2.53166
Im G_1	-0.48336	-1.13023	-1.13001	-3.88726	-3.92990	-3.92990
$d\sigma/dT$	1.1184×10^6	1.1252×10^6	1.1252×10^6	1.3248×10^5	1.3228×10^5	1.3228×10^5

b. $Z = 92 - \beta = 0.97$ ($T_0 = 3.0$)						
No Terms	$T = 50$ keV			$T = 150$ keV		
	30	100	250	30	100	250
Re F_1	-2.62557	-2.03047	-2.02502	0.36344	-0.01404	-0.01406
Im F_1	-28.50285	0.60027	0.59835	-0.44164	-0.03884	-0.03883
Re G_1	-13.33932	-12.84175	-12.82090	-5.28019	-7.96891	-7.96992
Im G_1	-206.866	9.27594	9.23053	-1.55627	-1.29747	-1.29659
$d\sigma/dT$	1.8138×10^5	9.5702×10^5	9.5685×10^5	1.0380×10^5	1.0801×10^5	1.0802×10^5

TABLE 2. COMPARISON OF $|\Delta Z|$ CHARGE CORRECTIONS TO THE FIRST BORN APPROXIMATION BASED UPON THE SECOND BORN APPROXIMATION (B2) AND MOTT PHASE-SHIFT CALCULATION (PS).

(T_0 is the approximation incident nucleus kinetic energy in units of GeV/nucleon. Charge corrections shown are rounded to the nearest integer.)

$\beta \backslash Z$	26		52		60		75		92		104	
	B2	PS	B2	PS	B2	PS	B2	PS	B2	PS	B2	PS
0.90 ($T_0=1.212$)	1	1	3	3	4	4	6	4	8	4	11	3
0.92 ($T_0=1.457$)	1	1	3	3	4	3	5	4	8	3	10	3
0.95 ($T_0=2.068$)	1	1	3	2	3	2	5	3	7	3	8	3
0.97 ($T_0=3.0$)	-	-	2	2	2	2	4	2	5	2	7	2

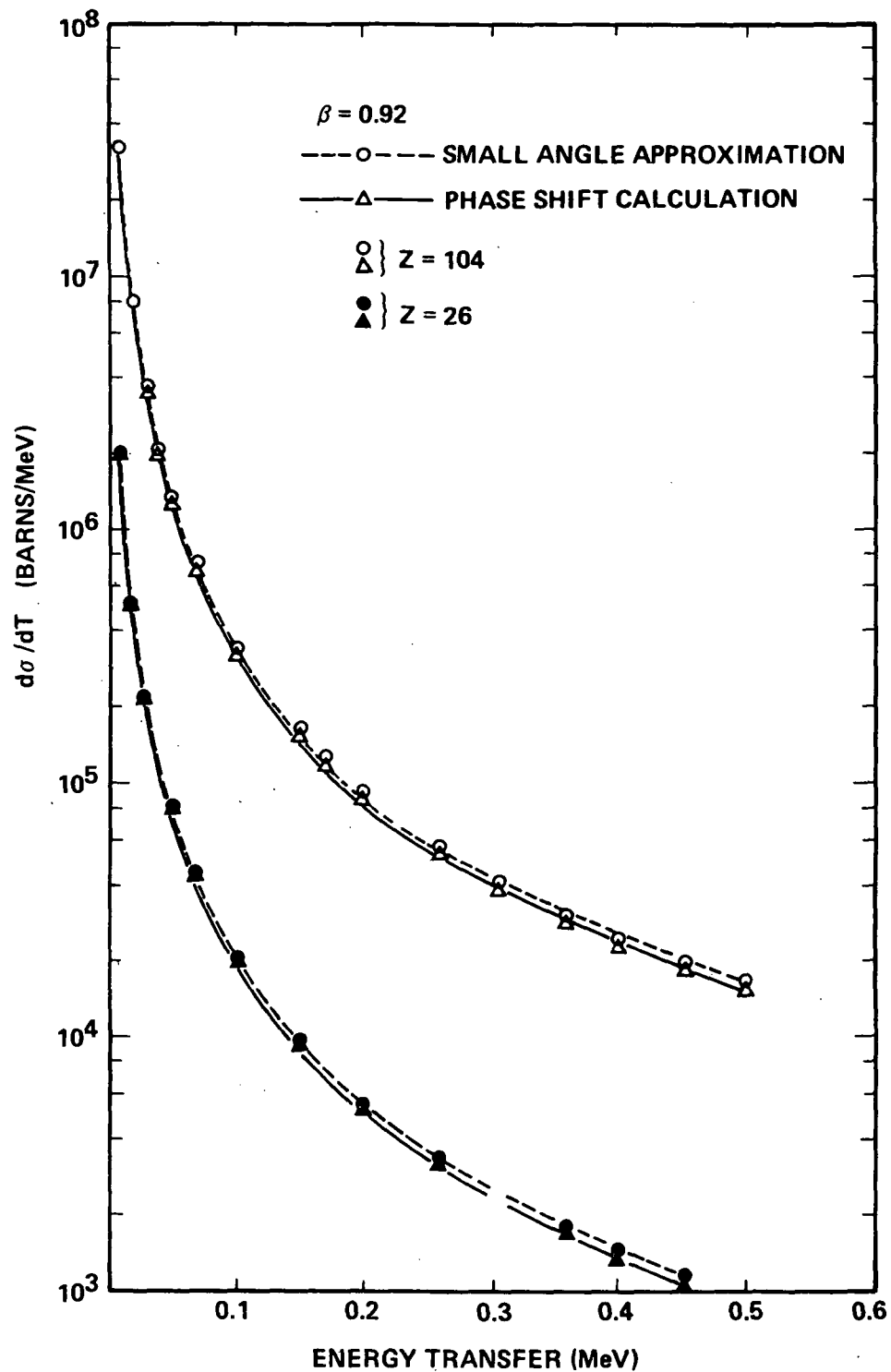


Figure 1. Comparison between small angle approximation and phase-shift calculation with 100 terms included for the phase-shift calculation. (Incident nuclei have charges $Z = 26$ and $Z = 104$ with $\beta = 0.92$.)

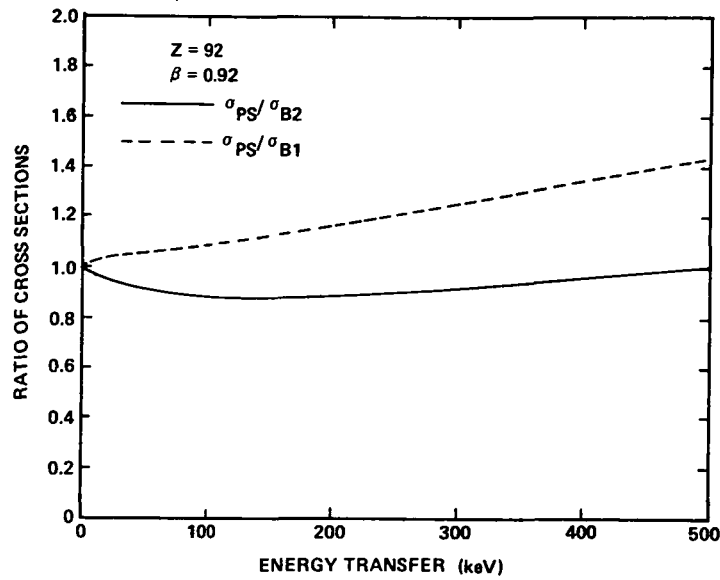


Figure 2. Comparison between ratios of phase-shift calculation to (1) second Born Approximation and (2) first Born approximation. (Incident nucleus has charge $Z = 92$ with $\beta = 0.92$.)

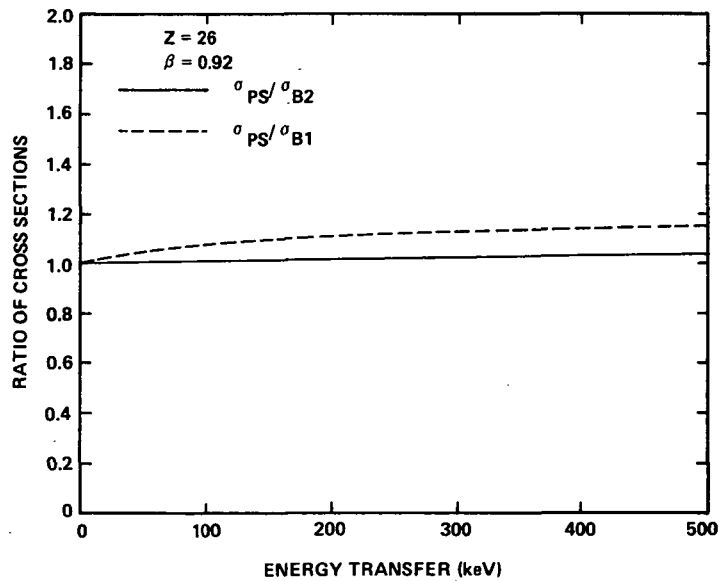


Figure 3. Comparison between ratios of phase-shift calculation to (1) second Born approximation and (2) first Born approximation. (incident nucleus has charge $Z = 26$ with $\beta = 0.92$.)

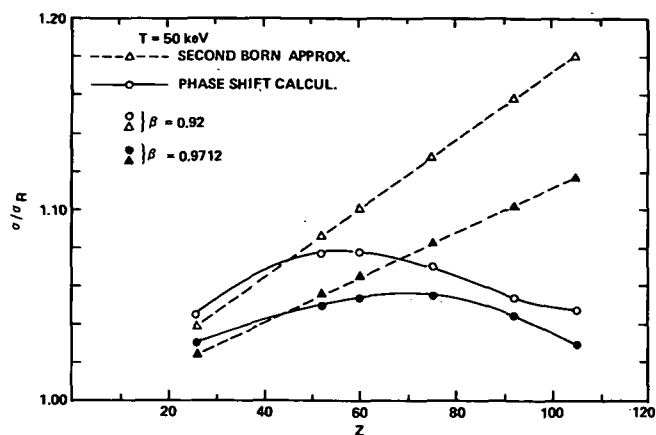


Figure 4. Ratio of phase-shift cross section and Rutherford cross section as function of Z for $\beta = 0.92$ and $\beta = 0.9712$, with ratio of second Born approximation to Rutherford cross section included for reference. (The energy transfer is 50 keV.)

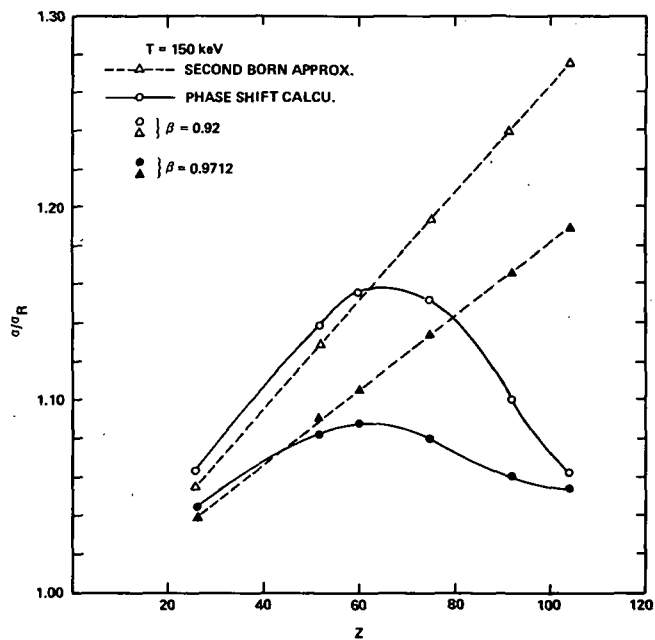


Figure 5. Ratio of phase-shift cross section and Rutherford cross section as function of Z for $\beta = 0.92$ and $\beta = 0.9712$, with ratio of second Born approximation to Rutherford cross section included for reference. (The energy transfer is 150 keV.)

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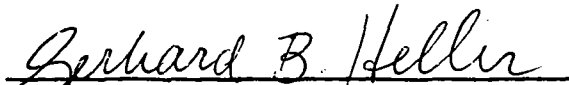
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